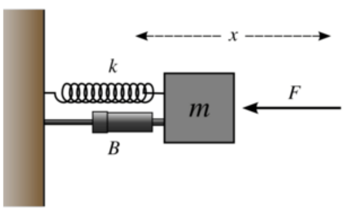
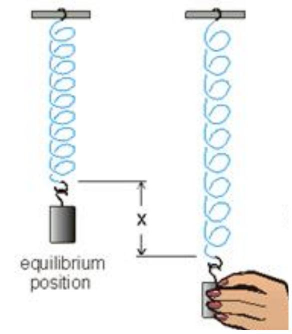
Philadelphia University

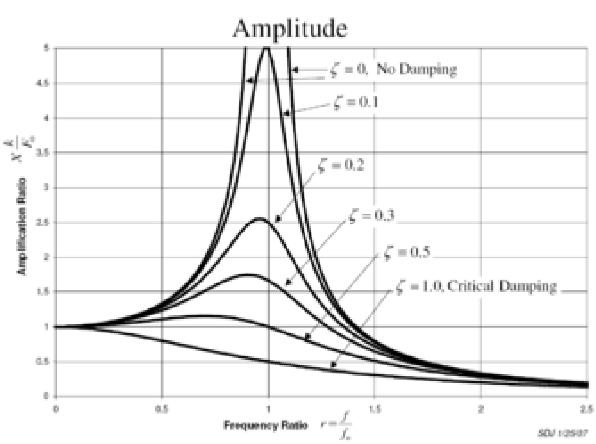
Faculty of Engineering

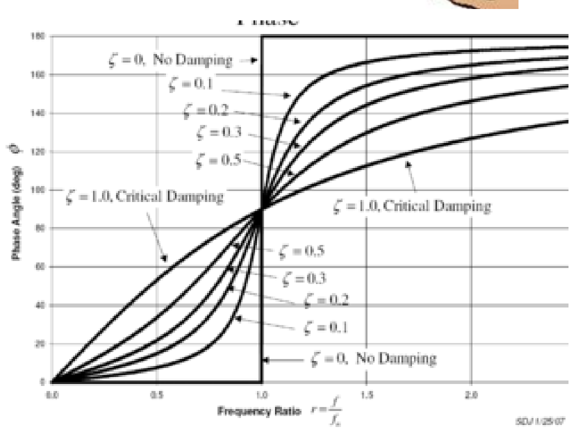
Mechanical Engineering Department

Mechanical Vibration Lab









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Vibration Review

➢ ELEMENTARY PARTS OF VIBRATORY SYSTEMS

Vibratory systems comprise means for storing potential energy (spring), means for storing kinetic energy (mass or inertia), and means by which the energy is gradually lost (damper). The vibration of a system involves the alternating transfer of energy between its potential and kinetic forms. In a damped system, some energy is dissipated at each cycle of vibration and must be replaced from an external source if a steady vibration is to be maintained. Although a single physical structure may store both kinetic and potential energy, and may dissipate energy, this chapter considers only *lumped parameter systems* composed of ideal springs, masses, and dampers wherein each element has only a single function. In translational motion, displacements are defined as linear distances; in rotational motion, displacements are defined as angular motions.

➢ TRANSLATIONAL MOTION

Spring:- In the linear spring shown in Figure1, the change in the length of the spring is proportional to the force acting along its length:

$$F = k(x - u) \tag{1}$$

The ideal spring is considered to have no mass; thus, the force acting on one end is equal and opposite to the force acting on the other end. The constant of proportionality k is the *spring constant or stiffness*.

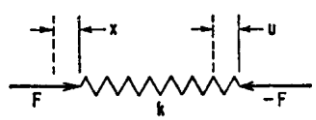


Figure1. Linear spring.

Mass:- A mass is a rigid body (Figure 2) whose acceleration $\ddot{x}$ according to Newton's second law is proportional to the resultant F of all forces acting on the mass:*

$$F = m\ddot{x} \tag{2}$$

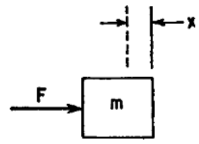


Figure 2. Rigid mass.

Damper. In the viscous damper shown in Figure 3, the applied force is proportional to the relative velocity of its connection points:

$$F = c(\dot{x} - \dot{u}) \qquad (3)$$

The constant c is the *damping coefficient,* the characteristic parameter of the damper. The ideal damper is considered to have no mass; thus the force at one end is equal and opposite to the force at the other end.

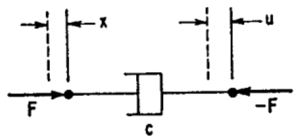


Figure.3 Viscous damper.

- **ROTATIONAL MOTION**

The elements of a mechanical system which moves with pure rotation of the parts are wholly analogous to the elements of a system that moves with pure translation. The property of a rotational system which stores kinetic energy is inertia; stiffness and damping coefficients are defined with reference to angular displacement and angular velocity, respectively. The analogous quantities and equations are listed in Table.1.

TABLE 1. Analogous Quantities in Translational and Rotational Vibrating Systems

Translational quentity	**rototinal quantity**
Linear displacement x	Angular displacement α
Force F	Torque M
Spring constant k	Spring constant k_r
Damping constant c	Damping constant c_r
Mass m	Moment of inertia I
Spring law $F = k(x_1 - x_2)$	Spring law $M = k_r(\alpha_1 - \alpha_2)$
Damping law $F = c(\dot{x}_1 - \dot{x}_2)$	Damping law $M = c_r(\dot{\alpha}_1 - \dot{\alpha}_2)$
Inertia law $F = m\ddot{x}$	Inertia law $M = I\ddot{\alpha}$

In as much as the mathematical equations for a rotational system can be written by analogy from the equations for a translational system, only the latter are discussed in detail. Whenever translational systems are discussed, it is understood that corresponding equations apply to the analogous rotational system, as indicated in Table.1.

➢ PERIODIC MOTION

Vibration is a periodic motion, or one that repeats itself after a certain interval of time called the period, *T*. Figure 4. illustrated the periodic motion time-domain curve of a steam turbine bearing pedestal. Displacement is plotted on the vertical, or *Y*-axis, and time on the horizontal, or *X*-axis. The curve shown in Figure 5 is the sum of all vibration components generated by the rotating element and bearing-support structure

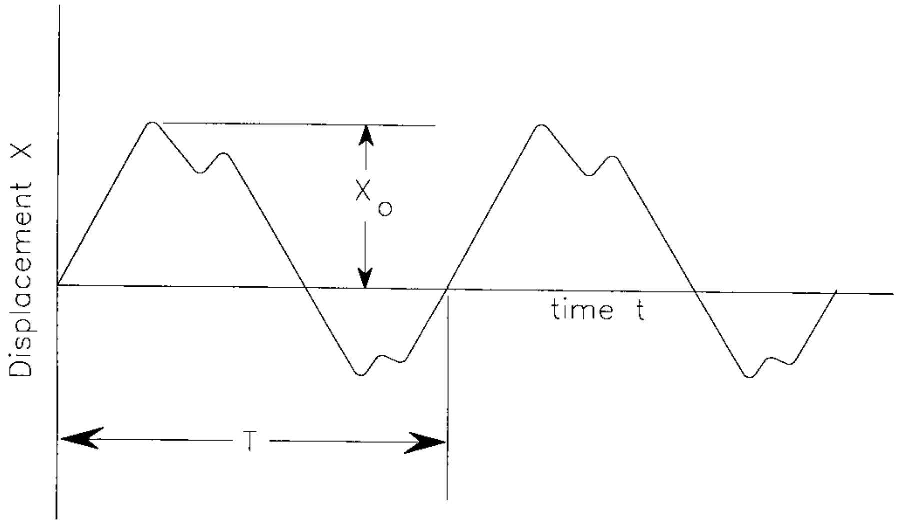


Figure 4. Periodic motion for bearing pedestal of a steam turbine.

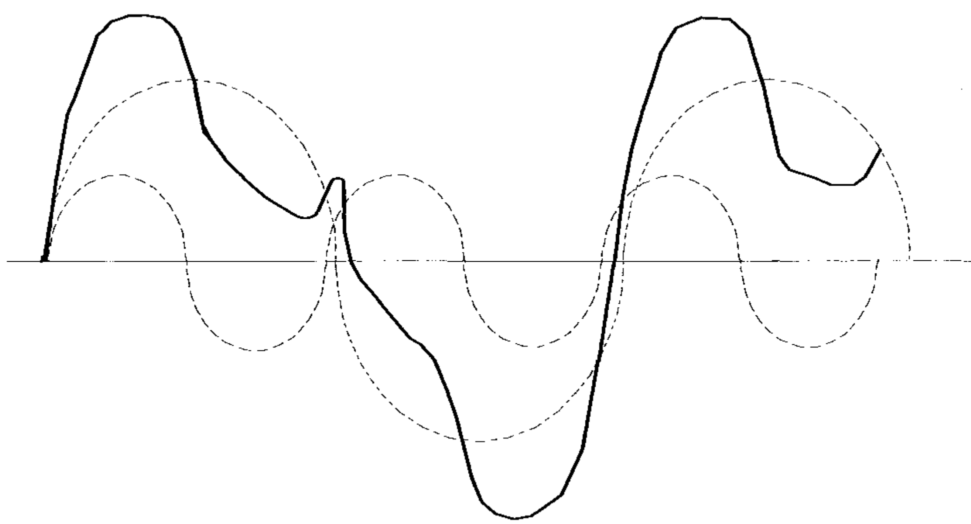


Figure 5 Discrete (harmonic) and total (none-harmonic) time-domain vibration curves.

➢ MEASURABLE PARAMETERS

As shown previously, vibrations can be displayed graphically as plots, which are referred to as vibration profiles or signatures. These plots are based on measurable parameters (i.e., frequency and amplitude). Note that the terms *profile* and *signature* are sometimes used interchangeably by industry. In this module, however, *profile* is used to refer either to time-domain (also may be called time trace or waveform) or frequency-domain plot.

Frequency

Frequency is defined as the number of repetitions of a specific forcing function or vibration component over a specific unit of time. Take for example a four-spoke wheel with an accelerometer attached. Every time the shaft completes one rotation, each of the four spokes passes the accelerometer once, which is referred to as four cycles per revolution. Therefore, if the shaft rotates at 100 rpm, the frequency of the spokes passing the accelerometer is 400 cycles per minute (cpm). In addition to cpm, frequency is commonly expressed in cycles per second (cps) or Hertz (Hz).

Amplitude

Amplitude refers to the maximum value of a motion or vibration. This value can be represented in terms of displacement (mils), velocity (inches per second), or acceleration (inches per second squared), each of which is discussed in more detail in the following section on Maximum Vibration Measurement.

Displacement

Displacement is the actual change in distance or position of an object relative to a reference point and is usually expressed in units of mils, 0.001 inch. For example, displacement is the actual radial or axial movement of the shaft in relation to the normal centerline usually using the machine housing as the stationary reference. Vibration data, such as shaft displacement measurements acquired using a proximity probe or displacement transducer should always be expressed in terms of mils, peak – peak

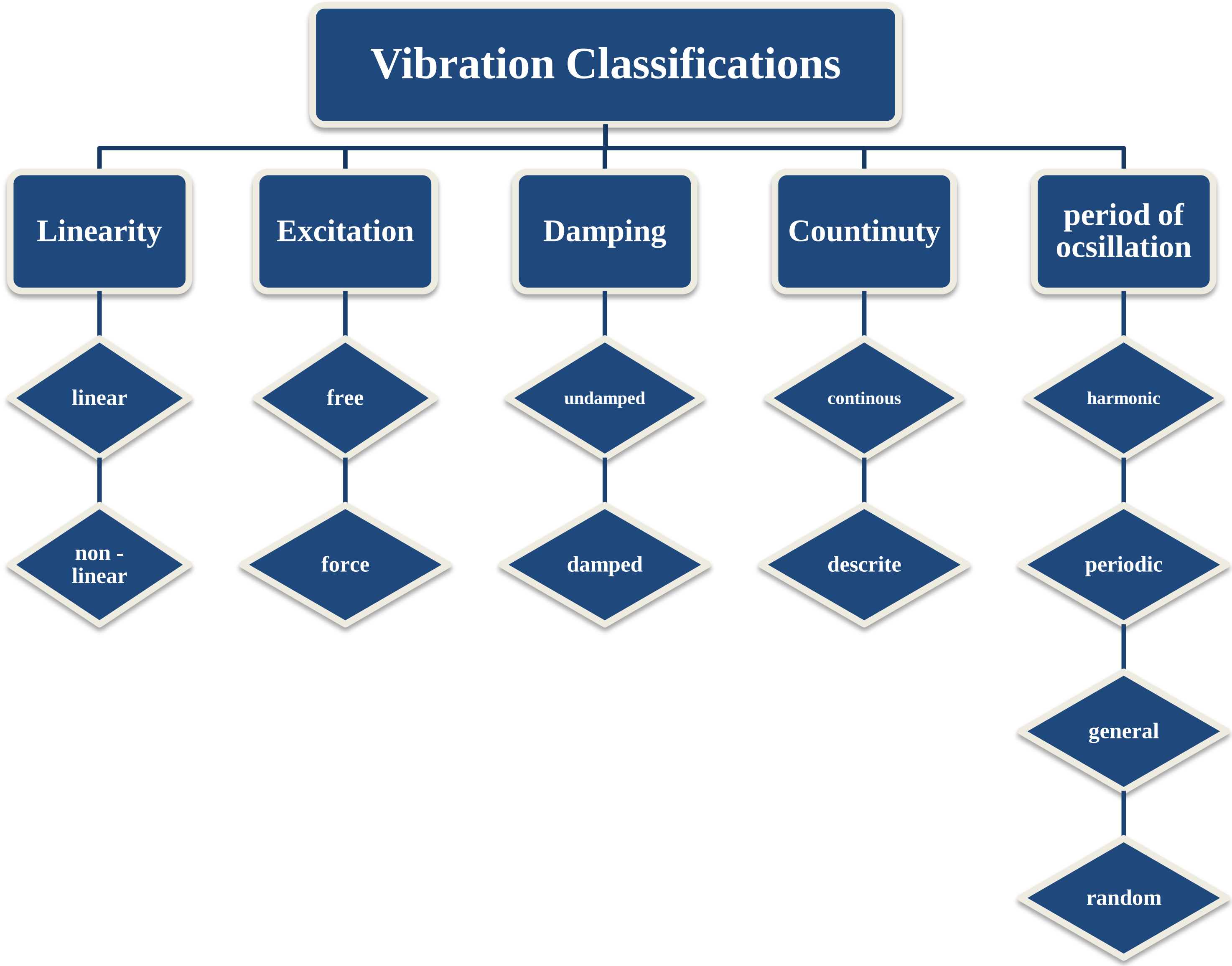
Velocity

Velocity is defined as the time rate of change of displacement (i.e., the first derivative, $\frac{dX}{dt}$ or $\dot{X}$) and is usually expressed as inches per second (in./sec). In simple terms, velocity is a description of how fast a vibration component is moving rather than how far, which is described by displacement.

Acceleration

Acceleration is defined as the time rate of change of velocity (i.e., second derivative of displacement, $\frac{d^2X}{dt^2}$ or $\ddot{X}$) is expressed in units of inches per second squared (inch/ sec^2) Acceleration is commonly expressed in terms of the gravitational constant, *g*, which is 32.17 ft/sec^2. In vibration analysis applications, acceleration is typically expressed in terms of *g*-RMS or *g*-PK. These are the best measures of the force generated by a machine, a group of components, or one of its components.

TYPES OF VIBRATION



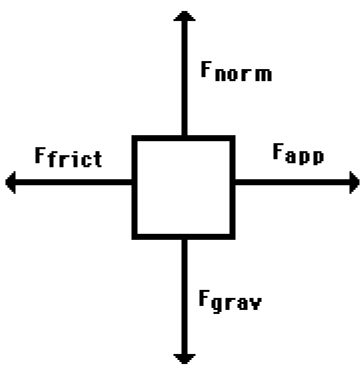
➢ Newtown Laws of motion

Newton's First Law of Motion:
Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.

Newton's Second Law of Motion:
The relationship between an object's mass *m*, its acceleration a, and the applied force *F* is $F = ma$. Acceleration and force are vectors; in this law the direction of the force vector is the same as the direction of the acceleration vector.

Newton's Third Law of Motion:
For every action there is an equal and opposite reaction.

Free Body Diagrams (F.B.D)



Drawing Free-Body Diagrams

Free-body diagrams are diagrams used to show the relative magnitude and direction of all forces acting upon an object in a given situation. A free-body diagram is a special example of the vector diagrams. These diagrams will be used throughout our study of physics. The size of the arrow in a free-body diagram is reflects the magnitude of the force. The direction of the arrow shows the direction which the force is acting. Each force arrow in the diagram is labeled to indicate the exact type of force. It is generally customary in a free-body diagram to represent the object by a box and to draw the force arrow from the center of the box outward in the direction which the force is acting. An example of a free-body diagram is shown at the right.

The free-body diagram above depicts four forces acting upon the object. Objects do not necessarily always have four forces acting upon them. There will be cases in which the number of forces depicted by a free-body diagram will be one, two, or three. There is no hard and fast rule about the number of forces which must be drawn in a free-body diagram. The only *rule* for drawing free-body diagrams is to depict all the forces which exist for that object in the given situation. Thus, to construct free-body diagrams, it is extremely important to know the various types of forces. If given a description of a physical situation, begin by using your understanding of the force types to identify which forces are present. Then determine the direction in which each force is acting. Finally, draw a box and add arrows for each existing force in the appropriate direction; label each force arrow according to its type. If necessary, refer to the list of forces and their description in order to understand the various force types and their appropriate symbols.

➢ Equations of motion:-

When a body is moving with a constant acceleration, the following relations are valid for the distance, velocity and acceleration.

$$
\begin{aligned}
v &= u + at && (1)\\
s &= \tfrac{1}{2}(u+v)t && (2)\\
s &= ut + \tfrac{1}{2}at^2 && (3)\\
v^2 &= u^2 + 2as && (4)\\
s &= vt - \tfrac{1}{2}at^2 && (5)
\end{aligned}
$$

By substituting (1) into (2), we can get (3), (4) and (5)

where

s = the distance between initial and final positions (displacement) (sometimes denoted *R* or *x*)
u = the initial velocity (speed in a given direction)
v = the final velocity
a = the constant acceleration
t = the time taken to move from the initial state to the final state

Report writing

Every student is required to submit his own separate report for each test conducted. Reports should be in hand-writing, on A4 paper. In general, the reports should be arranged in the following order:

1- Abstract

(An abstract is a self-contained, short, and powerful statement that describes a larger work. Components vary according to discipline. An abstract of a social science or scientific work may contain the scope, purpose, results, and contents of the work.)

2- Introduction

*(*Begin with background knowledge-What was known before the lab? What is the lab about? Include any preliminary/pre-lab questions. Also, include the purpose of the lab at the end of the introduction. Be clear & concise)

3- Materials and Equipment

(Can usually be a simple list, but make sure it is accurate and complete.)

4- Procedure

(Describe what was performed during the lab Using clear paragraph structure, explain all steps in the order they actually happened, If procedure is taken directly from the lab handout, say so! Do NOT rewrite the procedure!)

5- Collected Data

(Label clearly what was measured or observed throughout the lab Include all data tables and/or observation)

6- Calculations

(Show work, include units, and clearly label your results)

7- Results

(Are usually dominated by calculations, tables and figures; however, you still need to state all significant results explicitly in verbal form.)

8- Discussion and Analysis

(Answer any post-lab questions with complete thoughts. Assume the reader does not know anything about this topic.*)*

9- Conclusions

(Refer to the purpose- What was accomplished? Analyze your data, report your findings and include possible sources of error. How does this relate to topics outside of the classroom?)

10- References

Include an alphabetical list of all references used throughout the experiment and/or for writing the lab report. Include your textbook , lab manual, internet, etc.

DO NOT USE PERSONAL PRONOUNS!

(This includes: me, my, I, our, us, they, her, she, he, them, etc.)

Ex1.Mass-Spring System

I- Objectives:

1) To determine the stiffness of a helical spring using two methods;
   a -Deflection curve and *Hook's Law*.
   b -Time measurements.
   Then to compare their results with the analytical value.
2) To find the effective mass of the spring that has been used.
3) To evaluate the gravitational acceleration constant *g*.
4) To estimate the value of the modulus of rigidity *G* for the material of the helical spring, and compare it with the standard value for steel.

II- System Description:

The spring-mass system in *Figure1.1* shows an extension linear helical spring with an initial free length L_i, effective mass m_S, supported vertically from one of its ends; while the other end is free to elongate and attached to a load-carrier of *mass*. The free length of the spring loaded with the load carrier alone is L_o. Disks each of $(m_d = 0.4\ kg)$ mass are added to the carrier gradually, and each loading state causes the spring to elongate by the distance δ from its unloaded length L_o to get a total length of L.

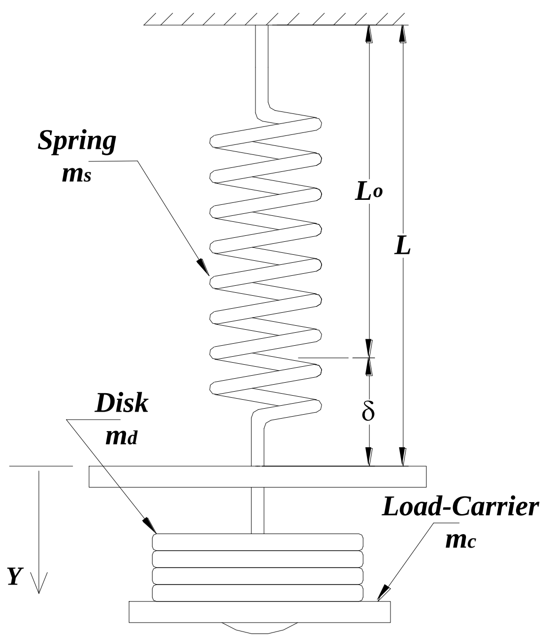


Figure 1.1 General layout of the experiment set-up

III- Governing Equations:

For the spring-mass system shown in *Figure-1.1*, in the case of free vibration in the vertical direction *Y*, the equation of motion of the system is given by:

$$M\ddot{y} + Ky=0 \tag{1}$$

where:

M is the total mass of the system, and equals to: $M = m + m_C + m_S$

m is the total mass of the disks: $m = \Sigma m_d$

From the equation of motion, we can find that:

* Natural frequency= $\omega_n = \sqrt{\dfrac{K}{M}}$ *(2)*

* Period of oscillation= $\tau = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{\dfrac{M}{K}} = 2\pi\sqrt{\dfrac{m + m_C + m_S}{K}}$ *(3)*

For the linear spring following *Hook's law*, then:

$$F_S = K\delta \tag{4}$$

But for the present system, the spring force F_S is also given by:

$$F_S = mg \tag{5}$$

Combine *eqns-4 & 5*, to get:

$$m = \frac{K}{g}\delta \tag{6}$$

For a helical spring, the stiffness is expressed analytically as:

$$K = \frac{Gd^4}{8ND^3} \tag{7}$$

IV- Experimental Procedures:

1. Hang the spring vertically with the load carrier attached to its end, and then measure the total length of the spring L_o. *(This length is not the initial free length of the spring L_i)*
2. Add one disk to the carrier $(m = m_d)$, and measure the total length of the spring after elongation L.
3. With this loading, stretch the spring downward, then leave it to oscillate freely and record the time needed to complete ten oscillations T.
4. Add another disk so that $(m = 2m_d)$, and repeat steps-2 & 3.
5. Continue by adding a disk each time for total ten disks $(m = 10m_d)$, and each time measure the parameters L and T.

V- Collected Data:

Table-1.1 Data collected from the experiment execution

Trial	*m (kg)*	*L (cm)*	*T (second)*
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Table-1.2 Dimensions and parameters of the spring

Parameter	*Value*
N (turns)	
D (mm)	
d (mm)	
L_o **(cm)**	

VI- Data Processing:

- **Square *eqn-3*, to get:** $\tau^2 = \frac{4\pi^2}{K}\left(m + m_C + m_S \right)$
- **Draw τ^2 versus *m* (call it figure 1.2)**
  1. Slope $S_1 = \frac{4\pi^2}{K} \Rightarrow K$ is determined.
  2. Intercept with the vertical axis $Y_{Inter} = \frac{4\pi^2}{K}\left(m_C + m_S\right) \Rightarrow m_S$ is determined.
  3. Intercept with the horizontal axis $X_{Inter} = -\left(m_C + m_S\right) \Rightarrow m_S$ is verified.
- **From *eqn-6*:** $m = \frac{K}{g}\delta$
- **Draw *m* versus δ (call it figure 1.3)**
  1. Slope $S_2 = \frac{K}{g} \Rightarrow K$ is also obtained.
  2. Multiply the slopes of the previous two steps. You get the value: $S_1 S_2 = \frac{4\pi^2}{g} \Rightarrow g$ is found, and compared to the standard value.
- **Use *eqn-7*:** $K = \frac{Gd^4}{8ND^3}$
  1. Find K directly.
  2. Compare the two experimental values of K obtained before, with this theoretical value.

VII- Results:

Table-1.3 Data processing analysis

Trial	*m (kg)*	*δ (mm)*	*τ (second)*	*τ^2 (second)^2*
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Table-1.4 *Data processing results*

Spring Stiffness K			
K (theoretical) = …………… (N/m)			
From:	*Slope*	*K (N/m)*	*Percent Error (%)*
Figure-1.2			
Figure-1.3			

Spring Effective Mass m_s			
From Figure-1.2:			
Y_{Inter} (kg.m/N)		*m_s (kg)*	
X_{Inter} (kg)		*m_s (kg)*	

Gravitational Acceleration g			
From Figures-1.2 & 1.3	*S_1S_2 (sec^2/m)*	*g (m/sec^2)*	*Percent Error (%)*

modulus of rigidity G			
From Figures-1.2	*slope (m/N)*	*G (Gpa)*	*Percent Error (%)*

VIII- Discussion and Conclusions:

- ***Answer the following questions:***
  1. In Eq.5 $F_S = mg$, why didn't we equate the spring force F_S with the total weight of the system Mg?
  2. What is the physical meaning of the Effective Mass of a spring? Is there an effective mass for Torsion springs?
  3. What is the relationship between the periodic time and the mass?
  4. If you use another spring with larger wire diameter, what is the effect on stiffness?
  5. If you use a compression spring instead of the tension spring with the same mass and geometry configuration, dose the stiffness remains the same or it will change? Why?
  6. You are an engineer in a scientific facility that tests the stiffness of manufactured springs, which one of the previous methods would you chose (static or dynamic) ? Why?

- ***From your own observations, mention the sources of errors in the experiment and suggest alternative procedures to reduce the errors*.**

- ***Design a similar experiment to find the torsion stiffness for a torsion spring.***

Ex2.Simple & Compound Pendulums

I- Introduction:

Simple pendulum is simply a concentrated mass *m* attached to one of the ends of a mass-less cord of length *l*, while the other end is fitted as a point of oscillation, such that the mass is free to oscillate about that fixed point in the vertical plane. The compound pendulum differs from the simple one in that it has a mass distribution along its length - that is its mass is not concentrated at a given point-, therefore it has a mass moment of inertia *I* about its mass centre.

Any rigid body that has a mass *m*, and mass moment of inertia *I* and suspended at a given distance *h* from its centre of gravity represents a compound pendulum.

It should be realised in the derivation of the governing equations, that the angle of oscillation of the pendulum, simple or compound, should be small.

II- Objectives:

This experiment aims at studying the behaviour of both simple and compound pendulums, in order to realise the following objectives:

1. The independence of the period of oscillation of the simple pendulum from its mass.
2. The relationship between the period of oscillation and its length.
3. The determination of the value of the gravitational acceleration *g*, to be compared with the known standard value.
4. Find the radius of gyration for a compound pendulum

III- System Description:

Part One- Simple Pendulum:

The schematic representation of the simple pendulum is shown in *Figure-2.1-a*, which consists of a small ball of mass m suspended by a mass-less cord of length l. The system is given an initial small angular displacement α, and as a result the pendulum oscillates in the vertical plane by a time varying angle $\theta(t)$ with the vertical direction.

Part Two- Compound Pendulum:

The compound pendulum is schematically shown in *Figure-2.2-b* below, and it consists of a uniform slender bar of total mass *m* and length *l*, which may be suspended at various points A along the bar with the aid of a sliding pivot situated at any distance h from the centre of gravity of the pendulum.

(For this case, the centre of mass is at the middle of the rod).

As a result of an initial angular displacement α the pendulum oscillates also with a time-varying angle $\theta(t)$ with the vertical direction.

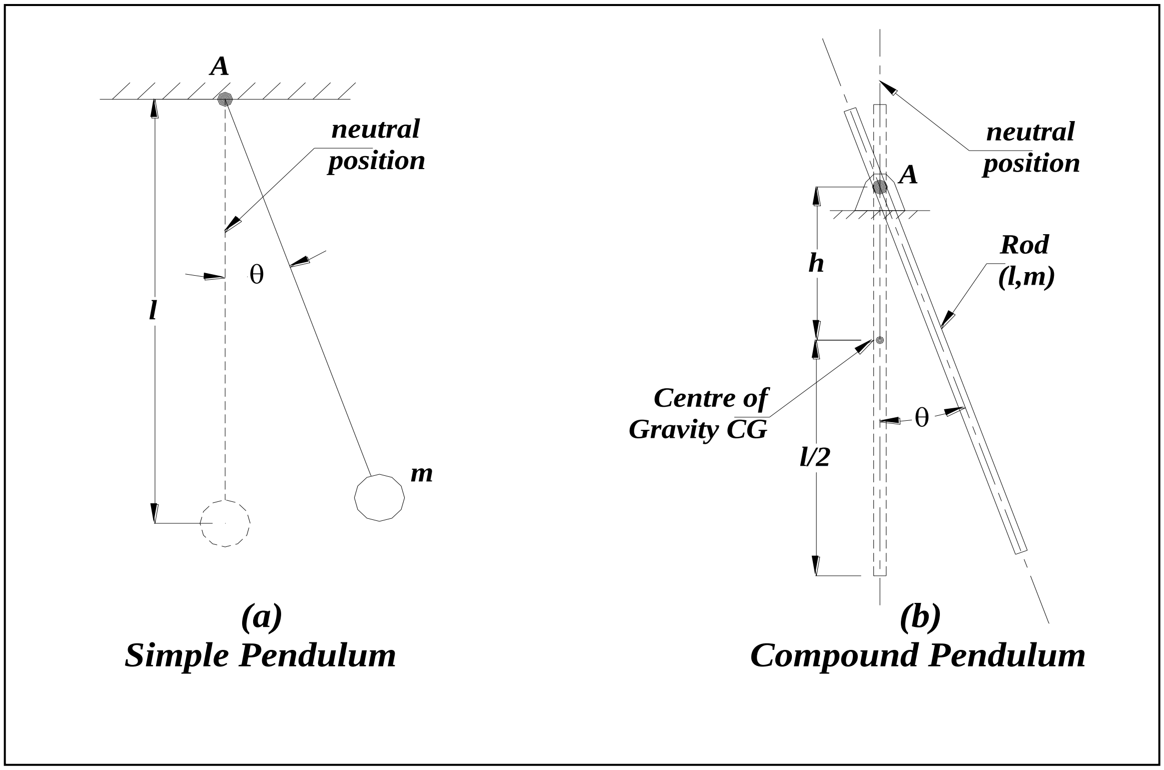


Figure-2.1 Schematic representation of the (a)simple pendulum (b)compound pendulum

IV- Governing Equations:

Part One - Simple Pendulum:

The dynamic equilibrium equation *(equation of motion)* corresponding to the tangential direction of motion of the concentrated mass yields:

$$mI\ddot{\theta} + mg\sin\theta = 0 \qquad (1)$$

Assuming small magnitude for the angle θ, so that $\sin\theta \approx \theta$, and simplifying *eqn-1* leads to the equation:

$$\ddot{\theta} + \frac{g}{l}\theta = 0 \qquad (2)$$

Let the motion defined by the function $\theta(t)$ be a simple harmonic motion defined as $\boldsymbol{\theta(t) = \alpha \sin \omega_n t}$, where ω_n is the natural frequency of the pendulum. Substituting for θ in *eqn-2* and simplifying gives ω_n as:

$$\omega_n = \sqrt{\frac{g}{l}} \tag{3}$$

The period of oscillation τ, is defined as the time required to complete one full cycle of motion or one oscillation. By observing the function $\theta(t)$, the period τ is given as:

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{l}{g}} \tag{4}$$

<u>Part Two- Compound Pendulum:</u>

For the compound pendulum, the dynamic equilibrium equation is obtained by taking the moments about pivot point *A* as given below:

$$I_A\ddot{\theta} + mg\sin\theta = 0 \tag{5}$$

where; I_A is the mass moment of inertia of the rod about the pivot point *A*.
Assuming small angle of oscillation and simple harmonic motion for $\theta(t)$, leads to the following expressions for the natural frequency ω_n and period τ, respectively:

$$\omega_n = \sqrt{\frac{mgh}{I_A}} \tag{6}$$

$$\tau = 2\pi\sqrt{\frac{I_A}{mgh}} \tag{7}$$

The mass moment of inertia about the pivot point I_A, is defined in terms of the mass moment of inertia about the centre of gravity I_{CG} and the distance *h* between the centre of gravity and the pivot point *A* as:

$$I_A = I_{CG} + mh^2 \tag{8}$$

or

$$I_A = m(K_{CG}^2 + h^2) \tag{9}$$

where; K_{CG} is the radius of gyration of the rod about the centre of gravity.

Using *eqns-7 & 9*, then the period of oscillation of the compound pendulum is given by the expression:

$$\tau = 2\pi\sqrt{\frac{K_{GC}^2 + h^2}{gh}} \tag{10}$$

V- Experimental Procedures:

Part One- Simple Pendulum:

Steel and plastic balls are used separately in this experiment as follows:

1. Attach the cord to the steel ball at one end, and attach the other end to the main frame. Record the length of the cord l.
2. Displace the ball form its neutral position by a small amount, and then release it to oscillate freely. Measure and record the time T required to complete ten oscillations.
3. Adjust the cord length to a new value and repeat step-2.
4. Repeat Step-3 six more times so that eight pairs of l and T are recorded.
5. Replace the steel ball with the plastic ball and repeat steps-1 through 4.

Part Two- Compound Pendulum:

The experimental procedures for the compound pendulum part are carried out through the following steps:

1. Measure and record the total length l of the rod. Since the rod is uniform, the geometrical centre point coincides with the rod's centre of gravity *CG*.
2. Pivot the rod at an arbitrary point *A*, and measure the distance from that point to the centre of gravity h. Displace the rod by a small angle from its neutral position and release it freely, then measure and record the time required to complete ten oscillations T.
3. Change the pivoting point *A* and repeat step-2.
4. Repeat step-3 eight more times so that ten pairs of h and T are recorded.

VI- Collected Data:

Part One- Simple Pendulum:

Table-2.1 *Collected data for the simple pendulum part*

Trial	***Steel Ball***		***Plastic Ball***	
	l (cm)	***T (second)***	***l (cm)***	***T (second)***
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Part Two- Compound Pendulum:

l = …………cm

Table-2.2 *Collected data for the compound pendulum part*

Trial	***h* (cm)**	***T* (second)**
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

VII- Data Processing:

Part One- Simple Pendulum:

- **Use *eqn-4*:** $\tau = 2\pi\sqrt{\frac{l}{g}}$

Evaluate the theoretical period τ_{Theor} corresponding to each length *l*.
The values of τ_{Theor} are to be compared with the experimental values τ_{Exper}.

- **Square both sides of *eqn-4* to get:** $\tau^2 = 4\pi^2 \frac{l}{g}$

- **Draw τ^2 versus l (call it Figure 2.2)**

Slope = $\frac{4\pi^2}{g}$ $\Rightarrow g$ is found and compared to the standard value.

Part Two- Compound Pendulum:

- **Square *eqn-10* and rearrange to get:** $\tau^2 h = \frac{4\pi^2}{g}\left(K_{CG}^2 + h^2\right)$
- **Draw τ^2h versus h^2 (call it Figure-2.3)**
    1. Slope = $\frac{4\pi^2}{g} \Rightarrow$ find g and compare it to the standard value.
    2. Intercept with the vertical axis $Y_{Int} = \left(\frac{4\pi^2}{g}\right)K_{CG}^2 \Rightarrow K_{CG}$ is obtained.
    3. Intercept with the horizontal axis $X_{Int} = -K_{CG}^2 \Rightarrow K_{CG}$ is verified.

- **Draw τ versus h (call it Figure-2.4.)**

- Find τ_{min} and the corresponding value of *h*.

VIII- Results:

Part One- Simple Pendulum:

Table-2.3 Data processing analysis for the simple pendulum part

Steel Ball					
Trial	***l*** **(cm)**	τ_{Exper} **(second)**	τ_{Theor} **(second)**	$(\tau_{Exper.})^2$ $(second)^2$	**τ Percent Error (%)**
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Table-2.4 Data processing analysis for the simple pendulum part

Plastic Ball					
Trial	***l*** **(cm)**	τ_{Exper} **(second)**	τ_{Theor} **(second)**	$(\tau_{Exper.})^2$ $(second)^2$	**τ Percent Error (%)**
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Table-2.5 Data processing results for the simple pendulum part.

Quantity	**Slope from Figure-2.2:**	**g (m/s^2)**	**Percentage Error of g (%)**
Steel Ball			
Plastic Ball			

Part Two- Compound Pendulum:

Table-2.6 Data processing analysis for the compound pendulum part.

Trial	**h (cm)**	**τ (second)**	**h^2 $(cm)^2$**	**$\tau^2 h$ ($cm.sec^2$)**
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Table-2.7 Data processing results for the compound pendulum part

From Figure-2.3		
Slope ($sec.^2/m$)	**g ($m^2/sec.$)**	**Percent Error (%)**
Y_{Int} ($sec^2.m$)	**K_{CG} (cm)**	
X_{Int} (m^2)	**K_{CG} (cm)**	**Percent Error (%)**

From Figure-2.4
τ_{min} (sec.)
h at $\tau = \tau_{min}$ (cm)

IX- Discussion And Conclusions:

- ***Answer the follwing questions:-***
  1. What do we mean by *"Simple Harmonic Motion" (SHM)*?
  2. Why did we use two masses with identical geometries for the simple pendulum experiment?
  3. What is the physical meaning of h being equal to zero? What is the corresponding period of oscillation?
  4. Why does the compound pendulum have the identity of possessing two values of h corresponding to the same period of oscillation τ?
  5. Based on the equation of motion, what is the difference between the simple and compound pendulums? How can we replace the compound pendulum with a simple pendulum having the same period of oscillation?

- ***From your own observations, mention the sources of errors in the experiment and suggest alternative procedures to reduce the errors*.**

- ***Mention some applications of both simple and compound pendelums in practical life.***
- ***Discuss the physical meaning of the radius of gyration and give examples for it is importance from practical life.***

- ***In this experimet, we use pendelums to find the gravitional accelertaion. Design another experiment with different proceduers for the same perpouce.***

Ex3.Mass Moment of Inertia Estimation-Part one: Bifilar Suspension

I- Introduction:

The Bifilar Suspension is a technique that could be applied to objects of different shapes, but capable to be suspended by two parallel equal-length cables, in order to evaluate its mass moment of inertia *I* about any point within the body.

In this experiment, the technique will be applied to find the mass moment of inertia of a regular cross-section steel beam about its centre of gravity.

II- Objectives:

This experiment is to be performed in order to evaluate the mass moment of inertia of a prismatic beam by introducing the method of Bifilar Suspension Technique.

III- System Description:

The layout of the experiment is shown schematically in Figure-3.1, in which we have a regular rectangular cross-section steel beam, of length L, total mass M, and mass moment of inertia about its centre of gravity I. The beam is suspended horizontally through two vertical chords, each of length l, and at distance b/2 from the middle of the beam CG.(Two small chucks are provided for attachment).

The system is initially balanced, and by exerting a small pulse in such a way that the beam keeps oscillating in the horizontal plane about its middle point (centre of gravity CG), then by virtue of the tension forces initiated in the suspension chords, the beam will oscillate making an angle θ with its neutral axis, and the suspension chords will make an angle ϕ with the original vertical position.

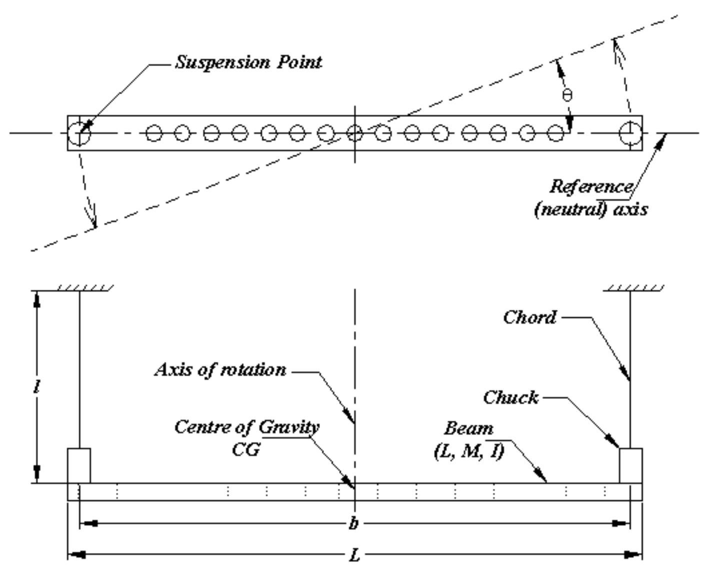


Figure-3.1 General layout of the Bifilar Suspension

IV- Governing Equations:

In the system shown in Figure-3.1, and under equilibrium conditions, the tension force in each chord is equal to Mg/2, and by disturbing the system with an initial angular displacement Θ about the middle point in the horizontal plane, it will oscillate with a time-varying angle θ(t) under the action of the tension forces in the chords.

Taking the summation of moments about the middle point *(Centre of Gravity CG)*, we get the equation of motion as:

$$\mathrm{I}\ddot{\theta}+\left(\frac{\mathrm{Mgb}}{2}\right)\emptyset = 0 \qquad (1)$$

But:

$\frac{b}{2}\theta = l\phi$ *(By equating the length of the arc of oscillation)*

Substituting in *eqn-1*, and rearranging:

$$\ddot{\theta} + \left(\frac{Mgb^2}{4Il}\right)\theta = 0 \qquad (2)$$

From the above equation of motion, we find that:

$$* \textit{ Natural frequency } = \omega_n = \sqrt{\frac{Mgb^2}{4Il}} \qquad (3)$$

$$* \textit{ Period of oscillation } = \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{4Il}{Mgb^2}} \qquad (4)$$

 # Analytical Solution:

Using the dimensions of the beam, then its mass moment of inertia about the centre of gravity can be found analytically as follows:

$$I = I\ (solid\ beam) - I\ (holes) + I\ (two\ chucks) = I_S - I_H + I_C \qquad (5)$$

$$I_S = \frac{M_S L^2}{12} = \frac{\rho w h L^3}{12} \qquad (6)$$

$$I_H = \frac{15}{2} M_H r^2 + 2M_H\ \Sigma X^2 = \rho \pi r^2 h\left(\frac{15}{2} r^2 + 2\ \Sigma X^2\right) \qquad (7)$$

$$I_C = M_C r_C^{\,2} + 2M_C\left(\frac{b}{2}\right)^2 = \rho\pi r_C^{\,2} h_C\left(r_C^{\,2} + \frac{b^2}{2}\right) \qquad (8)$$

Where:-

r: the radius of each hole.
X: the distance between the hole and the middle point of the beam.
r_C: the radius of the chuck.
h_C: the height of the chuck.

The geometry and the definitions of the basic parameters of the system are provided in *Figure-3.1*.

V- Experimental Procedures:

1- Attach the first chord to the main frame and measure its length, then attach the second chord to the main frame with the same length as the first one. (The length to be measured and included in the calculations l should include both the chord's length and the chuck's height, see Figure-3.1)
2- Insert a slender rod through the middle hole of the beam, to provide as an axis of rotation for the beam.
3- Hold the slender rod in place and give the beam a small displacement from one of its ends in the transverse direction. The beam should oscillate in the horizontal plane only.
4- Measure the time elapsed to complete ten oscillations T.
5- Release the chords then re-attach them at another length l, and repeat steps-2, 3 & 4.
6- Repeat step-5 four more times to get total six pairs of l and T.

VI- Collected Data:

Basic Parameters:

Table-3.1 Dimensions to be used according to Figure-3.1

Parameter	***Value***	***Parameter***	***Value***
L (cm)		***r_c (mm)***	
w (mm)		***h_c (mm)***	
h (mm)		***R (mm)***	
b (mm)		***h_m (mm)***	
r (mm)			

Table-3.2 Data collected

Trial	*l (cm)*	*T (second)*
1		
2		
3		
4		
5		
6		
7		
8		

VII- Data Processing:

- Square *eqn-4* to get: $\tau^2 = \left(\frac{16\pi^2 I}{Mgb^2}\right)\boldsymbol{l}$
- **Draw τ^2 versus l (call it Figure-3.3)**

Slope $= \frac{16\pi^2 I}{Mgb^2} \Rightarrow I$ is determined.

VIII- Results:

M = ………… (kg).

Table-3.3 Data processing analysis for the Bifilar Suspension Technique part

Trial	*l (cm)*	*τ (second)*	*τ^2 (second^2)*
1			
2			
3			
4			
5			
6			
7			
8			

Table-3.4 Data processing results for the Bifilar Suspension Technique part

Quantity	*Slope (sec.^{2}/m)*	*I (kg.m^2)*
From Figure-3.3		

Analytical Solution:

Table-3.5 Analytical determination of the mass moment of inertia I

I_S ***(kg.m²)***	
I_H ***(kg.m²)***	
I_C ***(kg.m²)***	
$I = I_S - I_H + I_C$ ***(kg.m²)***	

IX- Discussion And Conclusions:

- ***Answer the follwoing questions:-***
  1- In the first part, what modifications should be done *(concerning the derivation of equation of motion)* in order to determine the mass moment of inertia about any point other than the middle point of the beam? Derive the equation of motion for this case.
  2- Referring to the derivation of the equation of motion for the beam, why is it important to keep the angle of oscillation of the beam small during the execution of the experiment? What is the basic assumption that is based on assuming a small angle of oscillation?

- ***From your own observations, mention the sources of errors in the experiment and suggest alternative procedures to reduce the errors***.

- ***Design other procedures to find the mass moment of inertia other than the used in this experiment.***

Ex4.Mass Moment of Inertia Estimation-Part two: Auxiliary Mass Method

I- Introduction:

In this experiment, the two identical masses will be added to the primary system discussed in the previous experiment to find the mass moment of inertia of a regular cross-section steel beam about its centre of gravity.

II- Objectives:

This experiment is to be performed in order to evaluate the mass moment of inertia of a prismatic beam by introducing the method the Auxiliary Mass.

Then the values obtained from the this method will be compared with the values obtained experimentally and analytically in the previous experiment.

III- System Description:

The layout of the experiment is shown schematically in Figure-4.1, in which we have a regular rectangular cross-section steel beam, of length L, total mass M, and mass moment of inertia about its centre of gravity I. The beam is suspended horizontally through two vertical chords, each of length l, and at distance b/2 from the middle of the beam CG.(Two small chucks are provided for attachment).

The system is initially balanced, and by exerting a small pulse in such a way that the beam keeps oscillating in the horizontal plane about its middle point (centre of gravity CG), then by virtue of the tension forces initiated in the suspension chords, the beam will oscillate making an angle θ with its neutral axis, and the suspension chords will make an angle ϕ with the original vertical position.

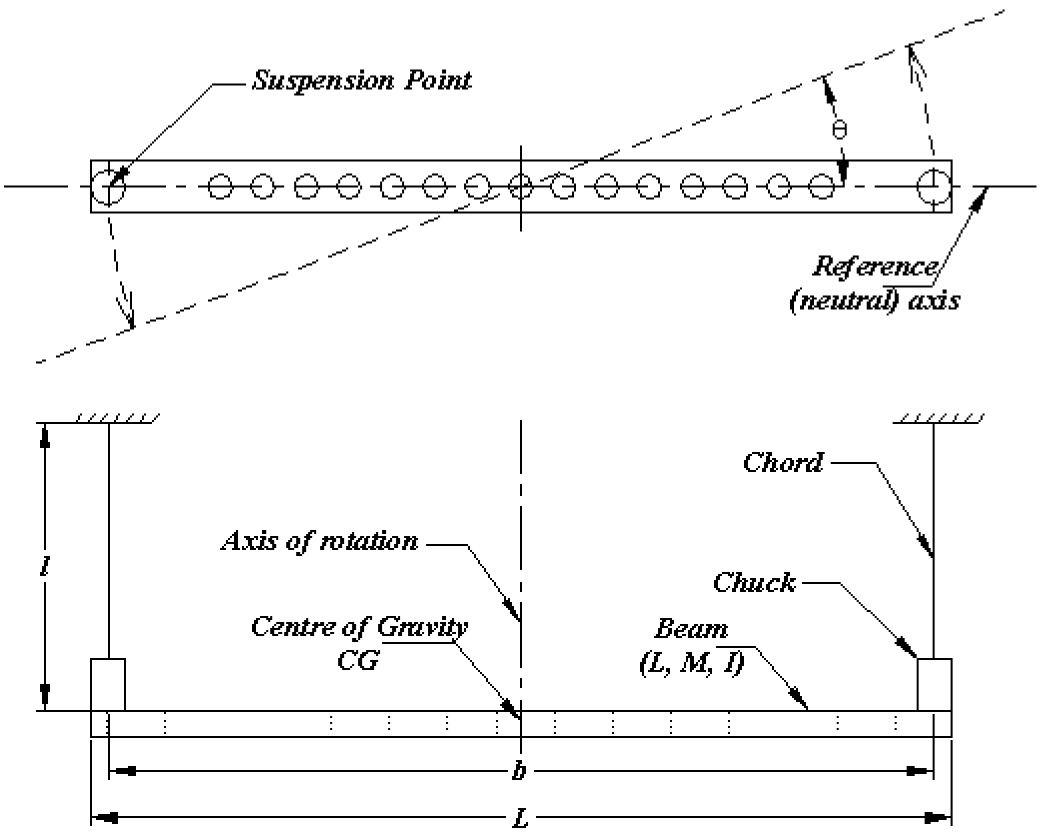


Figure-4.1 General layout of the Bifilar Suspension

IV- Governing Equations:

Consider the previous system with the addition of two identical circular disks of radius *R*, mass *m*, and inertia I_m; each at a side at distance *Y* from the middle of the beam. The resulting equation of motion of the modified system will be:

$$(I + 2I_m)\,\ddot{\theta} + \left(\frac{(M+2m)g\,b^2}{4l}\right)\theta = 0 \qquad (1)$$

Where:-

$I_m = m\,(R^2 + Y^2),\; m = \rho\pi R^2 h_m$

Rearrange *eqn-1*, yields:

$$\ddot{\theta} + \left(\frac{(M+2m)g\,b^2}{4l(I+I_m)}\right)\theta = 0 \qquad (2)$$

From *eqn-2*, the natural frequency and the period of oscillation are found as:

$$* \text{ Natural frequency } = \omega_n = \sqrt{\frac{gb^2(M+2m)}{4l(I+2I_m)}} \qquad (3)$$

$$* \text{ Period of oscillation } = \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{4l(I+2I_m)}{gb^2(M+2m)}} \qquad (4)$$

Analytical Solution:

Using the dimensions of the beam, then its mass moment of inertia about the centre of gravity can be found analytically as follows:

$$I = I\ (\text{solid beam}) - I\ (\text{holes}) + I\ (\text{two chucks}) \quad = I_S - I_H + I_C \qquad (5)$$

$$I_S = \frac{M_S L^2}{12} = \frac{\rho whL^3}{12} \qquad (6)$$

$$I_H = \frac{15}{2} M_H r^2 + 2M_H\ \Sigma X^2 = \rho\pi r^2 h\left(\frac{15}{2} r^2 + 2\ \Sigma X^2\right) \qquad (7)$$

$$I_C = M_C r_C{}^2 + 2M_C\left(\frac{b}{2}\right)^2 = \rho\pi r_C{}^2 h_C\left(r_C{}^2 + \frac{b^2}{2}\right) \qquad (8)$$

Where:-

r: the radius of each hole.
X: the distance between the hole and the middle point of the beam.
r_C: the radius of the chuck.
h_C: the height of the chuck.

The geometry and the definitions of the basic parameters of the system are provided in *Figure-4.1*.

V- Experimental Procedures:

1- Fix the examined system at any length l.
2- Put the two disks *(auxiliary masses)* at distance Y from the beam's middle point, each at a side, and record the value of Y.
3- Displace the beam slightly as in the previous part, and again measure the time elapsed in ten oscillations T.
4- Change the positions of the two masses to new value of Y, then repeat step-3.
5- Repeat step-4 for total different six values of Y.

VI- Collected Data:

Basic Parameters:

Table-4.1 Dimensions to be used according to Figure-4.1

Parameter	*Value*	*Parameter*	*Value*
L (cm)		***r_c (mm)***	
w (mm)		***h_c (mm)***	
h (mm)		***R (mm)***	
b (mm)		***h_m (mm)***	
r (mm)			

l = ……………(cm)
m = ……………(kg)

Table-4.2 Data collected for the Auxiliary Mass Method part

Trial	*Y (cm)*	*T (second)*
1		
2		
3		
4		
5		
6		

VII- Data Processing:

- **Square eqn-8 to get:** $\tau^2 = \dfrac{16\pi^2 l(I + 2I_m)}{gb^2(M + 2m)}$
- **Draw τ^2 versus I_m (call I Figure-4.4)**

1- Slope $= \dfrac{32\pi^2 l}{gb^2(M + 2m)}$ $\Rightarrow$ Determine g and compare it with the standard value.

2- Interception with the vertical axis $Y_{Int} = \dfrac{16\pi^2 Il}{gb^2(M + 2m)}$ $\Rightarrow$ I is determined.

3- Interception with the horizontal axis $X_{Int} = -\dfrac{I}{2} \Rightarrow$ I is verified.

VIII- Results:

Table-4.3 Data processing analysis for the Auxiliary Mass Method part

Trial	***Y (cm)***	I_m ***(kg.m²)***	τ^2 ***(second²)***
1			
2			
3			
4			
5			
6			

Table-4.4 Data processing results for the Auxiliary Mass Method part

From Figure-4.4			
Slope $(s^2/m^2.kg)$		***g*** $(m/sec.^2)$	
Y_{Int} $(sec.^2)$		I $(kg.m^2)$	
X_{Int} $(kg.m^2)$		I $(kg.m^2)$	

Analytical Solution:

Table-4.5 Analytical determination of the mass moment of inertia I

I_S $(kg.m^2)$	
I_H $(kg.m^2)$	
I_C $(kg.m^2)$	
$I = I_S - I_H + I_C$ $(kg.m^2)$	

Comparison:

Table-4.6 Comparison of I obtained by the two methods with the analytical value

Method:	I $(kg.m^2)$	***Percentage Error (%)***
Analytically		
Bifilar Suspension		
Auxiliary Mass(X_{int})		
Auxiliary Mass(Y_{int})		

IX- Discussion And Conclusions:

- ***Answer the follwoing questions:-***
  1. In the second part *(the Auxiliary Mass Method part)*, is it acceptable to use only one mass at either sides of the beam? Explain?

- ***From your own observations, mention the sources of errors in the experiment and suggest alternative procedures to reduce the errors***.

- ***Design other procedures to find the mass moment of inertia other than the used in this experiment.***

Ex.5 Forced Vibrations with Negligible Damping

I- Introduction:

Forced Vibrations is that mode of vibrations in which the system vibrates under the action of a time-varying force, generally; a harmonic external excitation of the form: $f(t) = F\sin(\omega t)$.

The importance of this mode rises in the practical field, as machines, motors and other industrial applications, exhibits this mode of vibrations, which may cause a serious damage of the machine.

II- Objectives:

In this experiment, we will apply both modes of vibrations; free and forced modes of vibrations, on a system in order to:

1- Evaluate of the natural frequency of the system using the following methods:
   1) Equation of motion.
   2) Time measurements.
   3) Drum speed.
   4) Resonance observation.

And the results of the various methods will be compared with the analytical value from the equation of motion.

2- Study the response of the system under the action of a time-varying force, then to determine and compare the magnification factor obtained both theoretically and experimentally.

III- System Description:

The system to be used in the experiment is shown in *Figure-5.1*, which consists of a regular rectangular cross-section beam of mass M_b, length L, width w and thickness t; pinned at one end to the main frame at point O, where it is free to rotate about, and suspended from point S by a linear helical spring of stiffness K at distance b from point O.

A motor with mass *($M = 4.55$ kg)* is fitted on the beam at distance a from pivot point O, and drives two circular discs with total eccentric mass m at distance e from the centre of the disc *(The eccentric mass is obtained from a hole in each disk with radius r and thickness t_d).* When the motor rotates these discs with speed ω, a harmonic excitation is established on the beam, and as a result of that, the beam vibrates in the vertical plane with angle $\theta(t)$ measured from the horizontal reference direction.

The free end of the beam carries a pencil that touches a rotating cylinder *(drum)* with a strip of paper covering it, so that you can draw the vibrations of the beam for a given period of time.

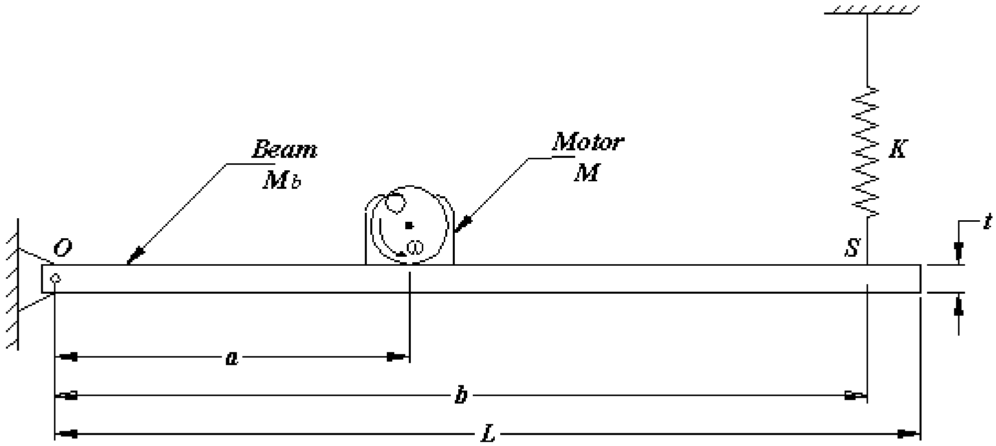


Figure-5.1 *General layout of the experiment set-up*

IV- Governing Equations:

Part One- Free Vibrations:

Referring to the system shown in *Figure-5.1*, with the motor is not operated; by giving the system an initial displacement and then leaving it to oscillate freely, the system will exhibit a free mode of vibrations, and the equation of motion in such case is obtained by taking the summation of moments about point *O* as follows:

$$I\ddot{\theta} + Kb^2\theta = 0 \qquad (1)$$

From which the natural frequency is found to be:

$$\omega_n = \sqrt{\frac{Kb^2}{I}} \qquad (2)$$

where:-

$$I = Ma^2 + M_b \frac{L^2}{3} \qquad (3)$$

$$K = \frac{Gd^4}{8ND^3} \quad \textit{(For a helical spring)} \qquad (4)$$

Also from time measurements, the natural frequency is equal to:

$$\omega_n = \frac{2\pi}{\tau} \tag{5}$$

in addition to getting the drum in touch with the pencil at the end of the beam, a graph of the oscillations of the beam can be obtained by rotating the drum. And so, we can say that:

$$\tau = \frac{C}{V} \tag{6}$$

where:-

C is the distance travelled per cycle.
V is the circumferential velocity of the drum.

And again, the natural frequency is obtained from *Eq.5.*

Part Two- Forced Vibrations:

When the motor is in operation, the beam will be imposed to a harmonic excitation due to the eccentric mass in each disk. This harmonic excitation will have the form:

$$f(t) = F\sin(\omega t) = me\omega^2 \sin(\omega t) \tag{7}$$

In this case, the equation of motion of the system is altered by:

$$I\ddot{\theta} + Kb^2\theta = ame\ \omega^2 \sin(\omega t) \tag{8}$$

Let $\theta(t) = \Theta\sin(\omega t)$, then the solution of the differential equation in *(8)* gives the amplitude of the angular displacement of the beam Θ as:

$$\Theta = \frac{mea\omega^2}{Kb^2 - I_A\omega^2} \tag{9}$$

And so, the vertical displacement of the end of the beam *Y* will be:

$$Y = L\Theta = \frac{meaL\omega^2}{Kb^2 - I_A\omega^2} \tag{10}$$

<u>Magnification Factor:</u>

Magnification Factor MF is the ratio between the dynamic amplitude of oscillation and the static amplitude of the same mode of displacement *(degree of freedom)*. And for this case, it is expressed as:

$$MF = \frac{Y_{Dynamic}}{Y_{Static}} \tag{11}$$

where:-

$\mathbf{Y_{Dynamic}}$, is given by *eqn-10* above.

$$Y_{Static} = \frac{meaL\omega^2}{Kb^2} \tag{12}$$

Substitute for $Y_{Dynamic}$ and Y_{Static} in *eqn-11*, and rearrange to get:

$$MF = \frac{1}{1-r^2} \tag{13}$$

where:-

$r = \dfrac{\omega}{\omega_n}$ *is the frequency ratio*.

V- Experimental Procedures:

1. Use the system described above while the motor is turned off, and give the beam a small vertical displacement, then release it to oscillate freely for ten oscillations. Record the elapsed time *T*.
2. Bring the drum in slight touch with the pencil at the end of the beam, after attaching the roll of paper to the drum, and then give the beam a small pulse to oscillate freely as before with the drum is held fixed.
3. Turn the motor of the drum on, and after ten seconds stop it and remove the chart for using it in the calculations.
4. Return to the original system by separating the drum from the pencil, and switch the motor on at a relatively slow speed.
5. Increase the speed of the motor slowly and notice the response of the system, and at the same time; try to identify the point at which resonance takes place *(When the largest amplitude of vibrations is noticed)*. Record the speed of the motor at that state N_r.
6. Attach the paper roll again to the drum, and make the pencil in touch with the drum. Activate the motor and set it to any desired speed *(Choose one that gives an appreciable amplitude of vibrations in the beam)*, and record that speed *N*.
7. Rotate the drum again for a while, and take the response curve obtained for the subsequent calculations.

VI- Collected Data:

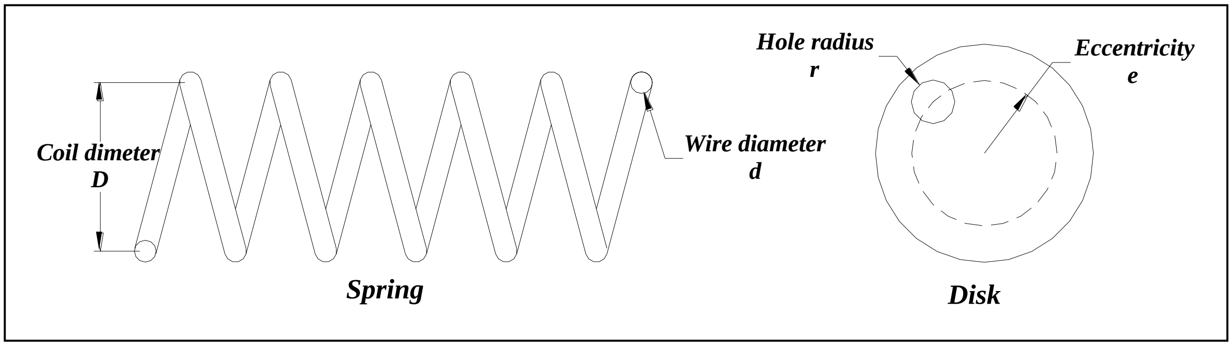


Figure-5.2 *Nomenclature of the coil spring and the rotating disc*

Basic Parameters And Dimensions:

Table-5.1 *Basic dimensions and parameters according to Figures-5.1& 2*

Beam			
Parameter	***Value***	***Parameter***	***Value***
L (cm)		***b (cm)***	
w (mm)		***t (mm)***	
Motor, Rotating Disks			
Parameter	***Value***	***Parameter***	***Value***
a (cm)		***r (mm)***	
e (mm)		***t_d (mm)***	
Spring			
Parameter	***Value***	***Parameter***	***Value***
D (mm)		***d (mm)***	
N (turns)			

Table-5.2 *Data collected from the experiment*

Free Vibrations Part	
Parameter	***Value***
T (second)	
C [from the first chart] (mm)	
V (m/s)	
Forced Vibrations Part	
Parameter	***Value***
Nr (rpm)	
N (rpm)	
A [amplitude of the second chart] (mm)	

VII- Data Processing:

Part One- Free Vibration:

- From the dimensions provided, and using *eqns-3 & 4*. Find M_b, I and K.
- Apply in *eqn-2* to find the theoretical natural frequency $\omega_{n\text{-}theor}$
- From T find τ, as: $\tau = \dfrac{T}{10}$
- From *eqn-5*, find ω_n.
- Compare it with $\omega_{n\text{-}theor}$.
- Calculate the velocity of the drum V, and use *eqn-6* to find τ.
- Apply again in *eqn-5* to find ω_n.
- Compare it with $\omega_{n\text{-}theor}$

Part Two- Forced Vibration:

- For the speed of the motor at resonance Nr, find the equivalent angular frequency of the motor ω.
- This frequency will be equal to the natural frequency of the system ω_n.
- Compare it with $\omega_{n\text{-}theor}$.
- From the value of N at which the second chart has been plotted, find the corresponding angular frequency ω.
  1. Evaluate the frequency ratio
     - r using $\omega_{n\text{-}theor}$, and apply *eqn-13* to evaluate MF.
  2. From *eqn-12*, find Y_{Static}, and from the second chart evaluate $Y_{Dynamic}$, then apply in *eqn-11* to evaluate MF.
  3. Compare the results of the two ways.

VIII- Results:

Table-5.3 *Data processing analysis*

Parameter	***Value***
M_b (kg)	
I ($kg.m^2$)	
K (N/m)	

Table-5.4 *Results of the natural frequency by the various methods*

Method	***Natural Frequency ω_n (rad/sec)***	***Percent Error (%)***
Analytical (E.O.M)		
Time Measurements		
Drum Speed		
Resonance Observation		

Table-5.5 *Magnification Factor MF results*

<table>
<tr><td rowspan="2">Methode-1</td><td>ω (rad/sec)</td><td>r ($\omega/\omega n$)</td><td>MF</td><td rowspan="2">Percent Error (%)</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td rowspan="2">Methode-2</td><td>$Y_{dynamic}$ (mm)</td><td>Y_{static} (mm)</td><td>MF</td><td rowspan="2"></td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

IX- Discussion And Conclusions:

- ***Answer the following questions:-***

  1. What is the meaning of the Static Amplitude of oscillation? In this case, derive the expression of *(Y_{static})* given in *eqn-12*?
  2. In the derivation of the equation of motion for the system, why did not we consider the effect of the gravitational forces *(weights of its components)* although they have moments about point *O*?
  3. For a practical system like a machine, suffering from such mode of vibrations, how could you modify its parameters (↑ or ↓), or add other components, in a way that minimises vibrations level?

- ***From your own observations, mention the sources of errors in the experiment and suggest alternative procedures to reduce the errors.***

- ***In this experiment, the unbalance causes the forced vibration. Mention other practical sources that causes forced vibration.***

- ***Discuss in your own language the concept of magnification factor and its relation to vibration analysis.***

Ex.6 Transverse Vibrations of a Beam

I- Objectives:

1) To introduce "*Dunkerley's Equation*", and demonstrate its use in studying transverse vibrations of beams.
2) To recognise the application of this equation on a simply supported beam, for the aim of:
   1- Determining the natural frequency ω_n of the simply supported beam, and then to compare it with the analytical value.
   2- Evaluation of its effective mass M_{Eff}, and then comparing it with the theoretical value.
   3- Determining the stiffness of the beam K, to be compared with the theoretical value.

II- System Description:

The system under study is shown in *Figure-7.1* below, which consists of a simply supported rectangular cross-section beam, of known dimensions L, w & t, modulus of elasticity E, total mass M_b and effective mass M_{Eff}.
Auxiliary masses *(disks)* M may be added to the system.
An electrical motor is fixed on the beam, and rotates a circular disk with eccentric mass to induce vibrations on the system.

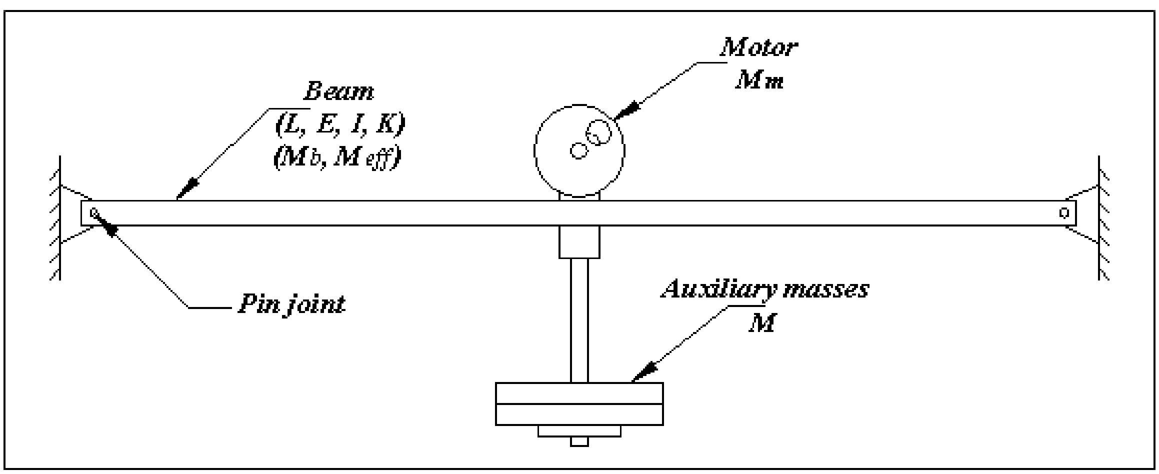


Figure-7.1 *General layout of the experiment set-up*

III- Governing Equations:

For the system shown in *Figure-7.1*, the equation of motion is given by:

$$(M + M_{Eff})\ddot{Y} + KY = 0 \qquad (1)$$

From which the natural frequency of the whole system ω_{ns} is found as:

$$\omega_{ns} = \sqrt{\frac{K}{M + M_{Eff}}} \qquad (2)$$

Square and expand this equation to get:

$$\frac{1}{\omega_{ns}^2} = \frac{M}{K} + \frac{M_{Eff}}{K} \Rightarrow \frac{1}{\omega_{ns}^2} = \frac{1}{\omega_{nm}^2} + \frac{1}{\omega_{nb}^2} \qquad (3)$$

This equation is known as the "**Dunkerley's Equation**",

Where:-

ω_{ns} is the natural frequency of the whole system.
ω_{nm} is the natural frequency of the motor.
ω_{nb} is the natural frequency of the beam.

Analytical Solution:

1. Natural Frequency (ω_{nb}):

Analytically, for a simply supported beam, an expression for the natural frequency ω_n can be derived to give:

$$\omega_n = \pi^2 \sqrt{\frac{EJ}{\rho A L^4}} = \pi^2 \sqrt{\frac{EJ}{M_b L^3}} \qquad (4)$$

2. Effective Mass (M_{Eff}):

The effective mass M_{Eff} of a simply supported beam is given in terms of its total mass M_b by the expression:

$$M_{Eff} = \frac{17}{35} M_b = 0.485714 M_b \qquad (5)$$

3. Stiffness (K):

The stiffness of simply supported beam is given as:

$$K = \frac{48EJ}{L^3} \tag{6}$$

Where:-

J is the polar moment of area and is found as: $J = \frac{bh^3}{12}$ where: b is the width of the beam and h is the thickness of the beam.

IV- Experimental Procedures:

1. Start with the system shown in *Figure-7.1* without any additional masses, and activate the motor to initiate vibrations on the beam.
2. Increase the speed gradually and observe the behaviour of the system, until you identify the resonance state where maximum amplitude of vibrations takes place, then record the speed of the motor N_R.
3. Add a (M) mass to the beam; and again, record the speed of the motor at resonance N_R.
4. Repeat step-3 another eight times to get total ten pairs of M and N_R.

V- Collected Data:

Table-7.1 *Dimensions of the beam*

Parameter	***Value***
L (cm)	
w (mm)	
t (mm)	

Table-7.2 *Data collected for the Dunkerley's Equation part*

Trial	***M (kg)***	***N_R (rpm)***
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

VI- Data Processing:

- For each value of N_R obtained, find the corresponding natural frequency for the system ω_{ns}.
- Draw $\left(\frac{1}{\omega_{ns}}\right)^2$ versus M, (call it Figure-7.2).

1) Slope = $\frac{1}{K} \Rightarrow K$ is determined.

Intercept with the vertical axis $Y_{Inter} = \left(\frac{1}{\omega_{nb}}\right)^2$ ω_{nb} is found.

Intercept with the horizontal axis $X_{Inter} = -M_{Eff} \Rightarrow$ Verify M_{Eff}.

- **Use *eqn-4* to find ω_{nb}**
- Compare it with the experimental values
- **From *eqn-5*, find M_{Eff}**

Compare it with the experimental value.

- **Determine *K* from *eqn-6***

Compare it with the experimental value.

VII- Results:

Table-7.3 Data processing analysis for the Dunkerley's Equation part

Theoretically:	
M_{Eff} (kg)	
K (N/m)	
ω_{nb} (rad/sec)	

Table-7.4 Data processing results for the Dunkerley's Equation part

From Figure-7.4		
Slope (m/N)	***K (N/m)***	***Percent Error (%)***
Y_{Inter} (sec/rad)^2	***ω_{nb} (rad/sec)***	***Percent Error (%)***
X_{Inter} (kg)	***M_{Eff} (kg)***	***Percent Error (%)***

VIII- Discussion and Conclusions:

- ***Answer the following questions:-***

1. Previously, both translational and rotational vibrations were examined. Mention the main differences between these types of vibration and the transverse vibration.
2. What is the relationship between the added mass and the natural frequency of the tested beam? Discus the physical meaning of this relation and how it can be used to control vibration levels.
3. In derivation of the mathematical model, what assumption been taken in consideration to transform the physical system to mass-spring model?

- ***From your own observations, mention the sources of error in this experiment and suggest alternative procedures to reduce it.***

- ***In this experiment, the observation of first resonance was used to determine the natural frequency of the whole configuration. Dose this approach is acceptable for this prepuce? Suggest another approach to find the natural frequency.***

- ***Dunkerley's Equation was and still an important to analyze the systems that contain multi-parts. Mention some of life applications that can be analyzed using this equation.***

Ex.7 Un-damped Dynamic Vibration Absorber

I- Objectives:

To demonstrate the principle of operation of the "***Un-damped Dynamic Vibration Absorber***" in eliminating vibrations of single degree of freedom systems.

II- System Description:

The Vibration Absorber is a secondary vibratory system attached to a primary one, such that it eliminates the vibrations of that primary system. One type of such absorbers is the *Un-damped Dynamic Vibration Absorber*, which is simply a spring-mass system.
Figure-7.1 below shows a form of such vibration absorbers; in which a cantilever beam having two identical masses at both ends -each at distance L_C- is fitted to the system used before and shown in *Figure-7.1* without the auxiliary masses.
The new system can be represented by a two-degrees of freedom system as the one shown schematically also in *Figure-8.1*, where:

M_1 is the mass of the primary system *(the beam and the motor)*.
M_2 is the mass of the secondary system *(each of the two suspended masses)*.
K_1 is the stiffness of the simply supported beam.
K_2 is the stiffness of the cantilever beam.

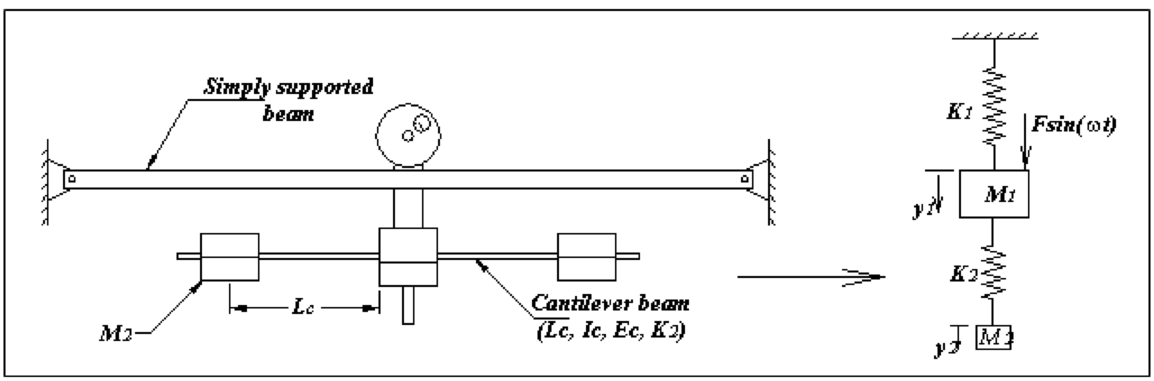


Figure-8.1 *General layout of the original system after the addition of the vibration absorber*

Taking each system separately *(primary & secondary)*, the equations of motion for the two systems are given by:

$$M_1\ddot{y}_1 + K_1 y_1 + K_2(y_1 - y_2) = F\sin(\omega t) \quad (1)$$

$$M_2\ddot{y}_2 + K_2(y_2 - y_1) = 0 \quad (2)$$

From which the steady state response is found for both as:

$$Y_1 = \frac{\left(K_2 - M_2\omega^2\right)F}{\left(K_1 + K_2 - M_1\omega^2\right)\left(K_2 - M_2\omega^2\right) \quad K_2^{\,2}} \tag{3}$$

$$Y_2 = \frac{K_2 F}{\left(K_1 + K_2 - M_1\omega^2\right)\left(K_2 - M_2\omega^2\right) \quad K_2^{\,2}} \tag{4}$$

But:

$$\delta_{Static} = \frac{F}{K_1} \tag{5}$$

So, *eqn-3* becomes:

$$\frac{Y_1}{\delta_{Static}} = \frac{1-\left(\frac{\omega}{\omega_{n1}}\right)^2}{\left[\left(1+\left(\frac{K_2}{K_1}\right)-\left(\frac{\omega}{\omega_{n1}}\right)^2\right)\left(1-\left(\frac{\omega}{\omega_{n2}}\right)^2\right)\right]-\frac{K_2}{K_1}} \tag{6}$$

Figure-8.2 below shows a graph of $\frac{Y_1}{\delta_{Static}}$ versus $\frac{\omega}{\omega_{n1}}$ for the primary system.

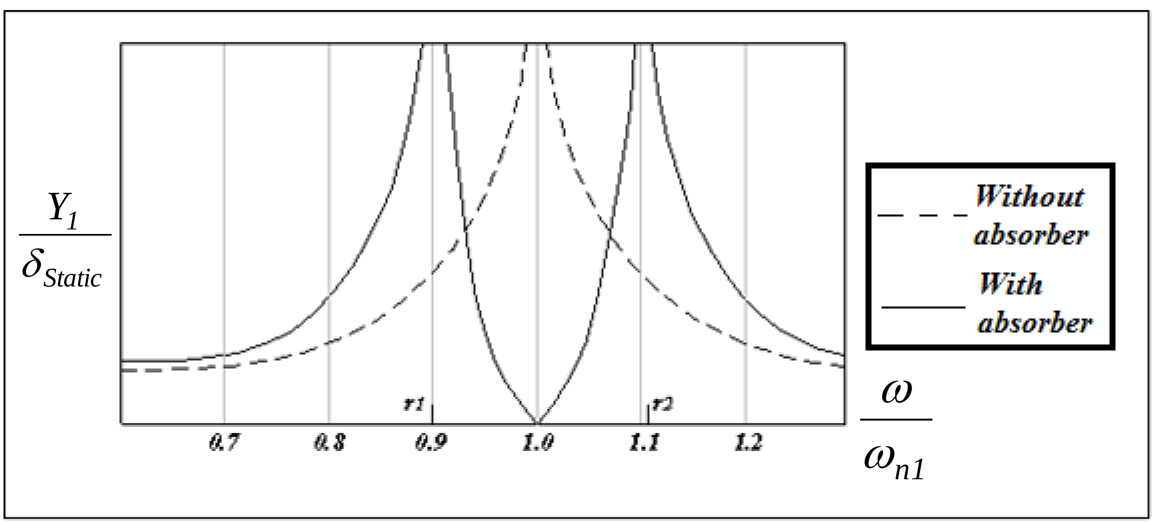


Figure-8.2. *Magnification factor versus frequency ration for the primary system*

Considering *eqns-3 & 6*, to eliminate the vibrations of the primary system, then:

$$Y_1 = 0 \Rightarrow K_2 - M_2\omega^2 = 0 \Rightarrow \omega^2 = \frac{K_2}{M_2}$$

But, at the state of resonance of the primary system:

$$\omega^2 = \omega_{n1}{}^2 = \frac{K_1}{M_1} \Rightarrow \frac{K_1}{M_1} = \frac{K_2}{M_2} \qquad (7)$$

That is, the natural frequency of the primary system should be equal to that of the secondary systems, and so:

$$\omega_R{}^2 = \frac{3E_C I_C}{M_2 L_C{}^3} \qquad (8)$$

To find the values of r_1 and r_2 in *Figure-8.2*, then:

$$Y_1 = \infty \Rightarrow \left(K_1 + K_2 M_1\omega^2\right)\left(K_2 M_2\omega^2\right) K_2{}^2 = 0$$

Define:

$$r = \frac{\omega}{\omega_n},\ R_M = \frac{M_2}{M_1} \Rightarrow r^4 - \left(2 + R_M\right) r^2 + 1 = 0 \text{ then:}$$

$$r_{1,2}{}^2 = \frac{2 + R_M \pm \sqrt{R_M{}^2 + 4R_M}}{2} \qquad (9)$$

From *eqn-9* we can find that:

$$\left\{\begin{array}{l} r_1 r_2 = 1 \\ r_1{}^2 + r_2{}^2 = 2 + R_M \end{array}\right\} \qquad (10)$$

IV- Experimental Procedures:

1. Run the motor at until the resonance occurs; then slide the two masses slowly on the cantilever beam by equal distances, until you detect the best sense of elimination of vibrations of the simply supported beam. Record the length L_C.

2. Keep the vibration absorber in the previous modified configuration, and run the motor at low speed. Increase the speed slowly, and determine the speed of the motor at each one of the two cases of resonance shown in *Figure*-8.2; that is, N_1 and N_2 corresponding to r_1 and r_2, respectively.

V- Collected Data:

Table-8.1 *Parameters of the cantilever beam and the suspended masses*

Parameter	***Value***
L_C ***(cm)***	
w_C ***(mm)***	
t_C ***(mm)***	
M_2 ***(kg)***	

Table-8.2 *Data collected for the Vibration Absorber part*

Parameter	***Value***
N_1 ***at*** r_1 ***(rpm)***	
N_2 ***at*** r_2 ***(rpm)***	

VI- Data Processing:

- Apply in *eqn-8*, with $\omega_n = \omega_{n1}$ to find L_C for the cantilever beam.
- Compare L_C calculated with that obtained experimentally.
- Use *eqn-9* to evaluate r_1 and r_2.
- Compare these values with those observed experimentally.
  Then verify your experimental results using *eqn-10*.

VII- Results:

Table-8.3 Data processing results for the Vibration Absorber part

Parameter	***Theoretical***	***Experimental***	***Percent Error (%)***
L_C ***(mm)***			
r_1			
r_2			

VIII- Discussion And Conclusions:

- ***Answer the follwing questions:-***

  1. How dose the vibration absorber control vibration level?
  2. After adding the absorber, two resonances were generated. Explain why?

- ***From your own observations, mention the sources of error in this experiment and suggest alternative procedures to reduce it.***

Ex.8 Static & Dynamic Balancing

I- Introduction:

Balancing is an essential technique applied to mechanical parts of rotational functionality *(wheels, shafts, flywheels…)*, in order to eliminate the detected irregularities found within it, and that may cause excessive vibrations during operation, and act as undesirable disturbances on the system being in use. Such irregularities may rise due to the inhomogeneous distribution of material within the part, bending and deflection of rotating shafts, and eccentricity of mass from the axis of rotation of the rotating disks and rotors.

These irregularities lead to small eccentric masses that disturb mass distribution of the part, and the last generate *centrifugal forces* when the part is in rotation; the magnitude of these forces increases rapidly with speed of rotation, and enhances vibrations level during operation, and cause serious problems.

II- Objectives:

This experiment is established in order to introduce and interpret the general features of balancing technique, in addition to familiarise the student with the basic steps in applying both static and dynamic balancing techniques on unbalanced mechanical parts.

III- Technique Presentation:

Part One- Static Balancing:

Static Balancing simply means the insurance of mass distribution about the axis of rotation of the rotating mechanical part in the radial directions, without consideration of that distribution in the axial *(longitudinal)* direction.

To illustrate this; consider a circular disk of perfect mass distribution, with the points *A* and *B* are at two opposite positions on the circumference of the disk, but each is on one of the faces of the disk, and suppose that a point mass with the same value is fixed at each of the two points *A* and *B*.
Generally, static balancing looks to the part in the direction of its axis of rotation, so in this case, as the two eccentric masses at *A* and *B* are in opposite positions with equal distances from the central axis, the disk is considered statically balanced although these masses are at different axial positions.

Practically, static balancing is performed by taking the part like a disk with its axis of rotation oriented horizontally, and rotating it several times; and at the end of each run after getting stable, a mark is made in the lower part of the disk on one of its faces. If the different marks are distributed randomly over the circumference of the disk, then the disk is of good mass distribution and considered balanced; but in the case that they accumulate in a small region, it is realised that there is a mass concentration in that part of the disk, and this can be treated either by taking small mass from there, or by adding mass to the opposite position of the disk.

Static Balancing Machine shown in *Figure-10.1* below is used for faster and more accurate static balancing operations. The machine is simply a pendulum, that is balanced and stable in a vertical configuration with no loading, and free to tilt in all directions about a ball joint; but when the pendulum is loaded with an unbalanced disk on its platform, it tilts by some angle from the original orientation. The side to which it tilts shows the position of the eccentric mass, and the angle by which it tilts θ is proportional to the magnitude of that eccentric mass to be compensated.

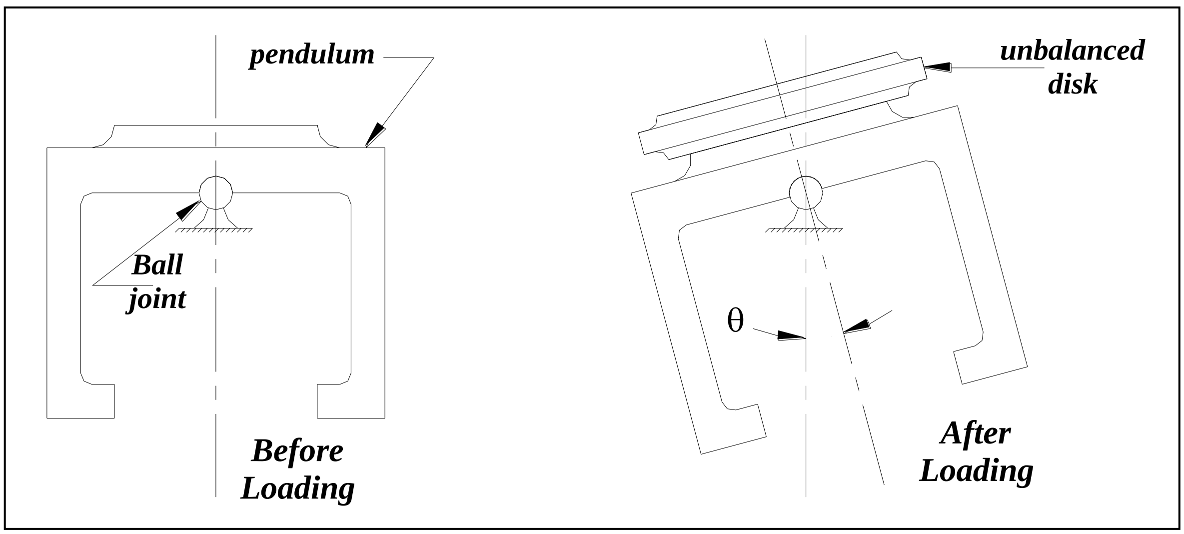


Figure-10.1 Schematic representation of the Static Balancing Machine

From the previous discussion, the only condition to be satisfied for static balancing to be achieved is that:-
"The resultant force of all the forces caused by the rotation of the out of balance masses, in a given rotating part should be zero", that is:

$$\Sigma \vec{F}_i = 0 \qquad (1)$$

The force F_i is given by:

$$Fi = m_i e_i \Omega^2 \qquad (2)$$

where; m_i is the out of balance mass *(eccentric mass)*.
e_i is the distance from axis of rotation *(eccentricity)*.
Ω is the angular speed of the part.

(Note: Eq.1 is a vector equation, in which each force is a vector of a magnitude given by Eq.2, and direction denoted by the angle θ_i, measured from the reference horizontal direction).

Part Two- Dynamic Balancing:

Dynamic Balancing differs from static balancing in that the mass distribution of the part is detected in all directions, and not only about the central axis; and so, not only the magnitude of the unbalanced mass and its distance from the axis of rotation are to be determined, but also its position in the axial *(longitudinal)* direction of the rotational part.

To illustrate the meaning of this, consider a disk rotating with an angular speed Ω, with different out of balance masses m_i, each with eccentricity e_i from the axis of rotation. These masses are not expected to be in the same plane, but in different locations along the disk's axial direction; in addition, each mass will produce a centrifugal force making an angle θ_i with the reference horizontal direction in its own plane.
The system described previously and shown schematically in *Figure-10.2*, can be easily treated by choosing any plane as the reference for the other planes containing the eccentric masses, such that each one of them is at distance a_i from that reference plane.
And for simplicity, choose plane-1 as the reference plane, where a_1 becomes zero.

Generally, for the dynamic balancing of a system to be achieved, then:
"The resultant force of all centrifugal forces caused by the out of balance masses should be zero *(as in static balancing)*, in addition to that the summation of their moments about any point should be also zero", that is:

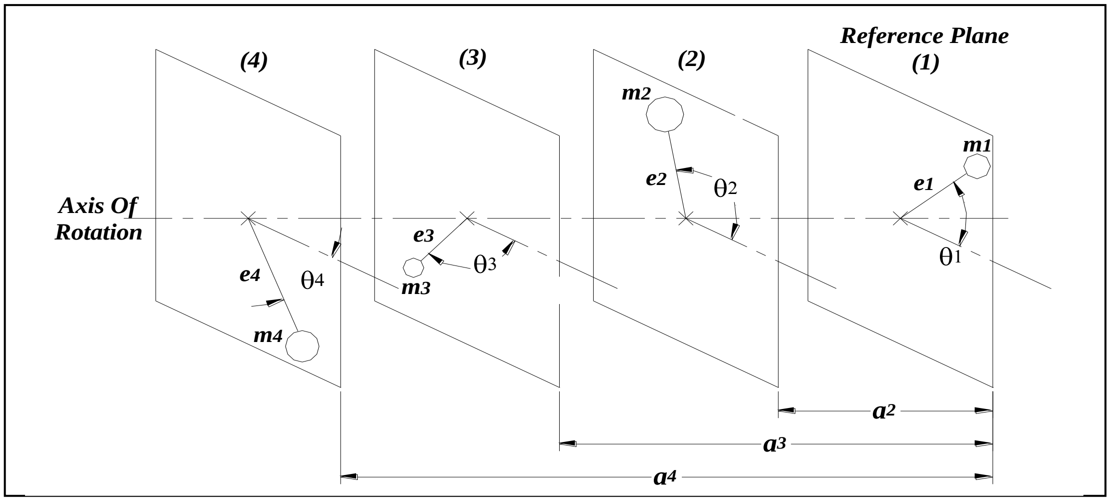


Figure-10.2 General case of a 3-D system to be dynamically balanced

$$\Sigma \vec{F}_i = 0 \qquad (1)$$

$$\Sigma \vec{M}_i = 0 \qquad (3)$$

And again, the forces in *eqn-1* are given by *eqn-2*, and the moments in *eqn-3* are given by:

$$Mi = a_i m_i e_i \Omega^2 \qquad (4)$$

And so, after choosing a reference plane, translate all the centrifugal forces in the other planes to that plane as forces $(m_i e_i \Omega^2)$ and moments $(a_i m_i e_i \Omega^2)$, and there you can apply the vector summation of forces and moments separately to satisfy the requirements of dynamic balancing mentioned in *eqns-1 & 3*.

IV- System Description:

The system we are dealing with is shown in *Figure-10.3*, which consists of four blocks with the same geometry and dimensions, but each has a different size hole and so different eccentric mass. The four blocks are spaced along a shaft driven by an electrical motor, where each is fixed at distances S_i from its end, with angle θ_i measured from the horizontal direction.
The electrical motor is attached to the shaft by a flexible belt, and provides the shaft with rotation at various speeds; The shaft and the four blocks are carried on a circular table, which is attached to the rigid frame by flexible mountings that permits the sense of vibrations during the operation of the system.

The system in hand is to be balanced using the principles outlined before. The dimensions of all the blocks are provided, while the angular orientation and the distance from the end of the shaft are given for the first two blocks only; and so, you have to find the missing parameters of the other two blocks analytically, such that balancing state is accomplished.

V- Governing Equations:

In this experiment, the major formulas to be used have been given in *eqns-1, 2, 3 & 4*; and according to the given system, *eqns-1 & 3* can be extracted to:

$$\sum \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

$$\Rightarrow m_1 e_1 \cos\theta_1 + m_2 e_2 \cos\theta_2 + m_3 e_3 \cos\theta_3 + m_4 e_4 \cos\theta_4 = 0 \qquad (5)$$

$$\Rightarrow m_1 e_1 \sin\theta_1 + m_2 e_2 \sin\theta_2 + m_3 e_3 \sin\theta_3 + m_4 e_4 \sin\theta_4 = 0 \qquad (6)$$

$$\sum \vec{M}_i = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4 = 0$$

$$\Rightarrow a_1 m_1 e_1 \cos\theta_1 + a_2 m_2 e_2 \cos\theta_2 + a_3 m_3 e_3 \cos\theta_3 + a_4 m_4 e_4 \cos\theta_4 = 0 \qquad (7)$$

$$\Rightarrow a_1 m_1 e_1 \sin\theta_1 + a_2 m_2 e_2 \sin\theta_2 + a_3 m_3 e_3 \sin\theta_3 + a_4 m_4 e_4 \sin\theta_4 = 0 \qquad (8)$$

To find the eccentric mass *m* and the eccentricity *e* for each block, then: According to *Figure-10.4* shown below, by assuming that the sector removed from the circle of diameter D_1 contributes approximately *90°* of the full circle, then the eccentric mass and its eccentricity can be expressed by the following formulas, respectively:

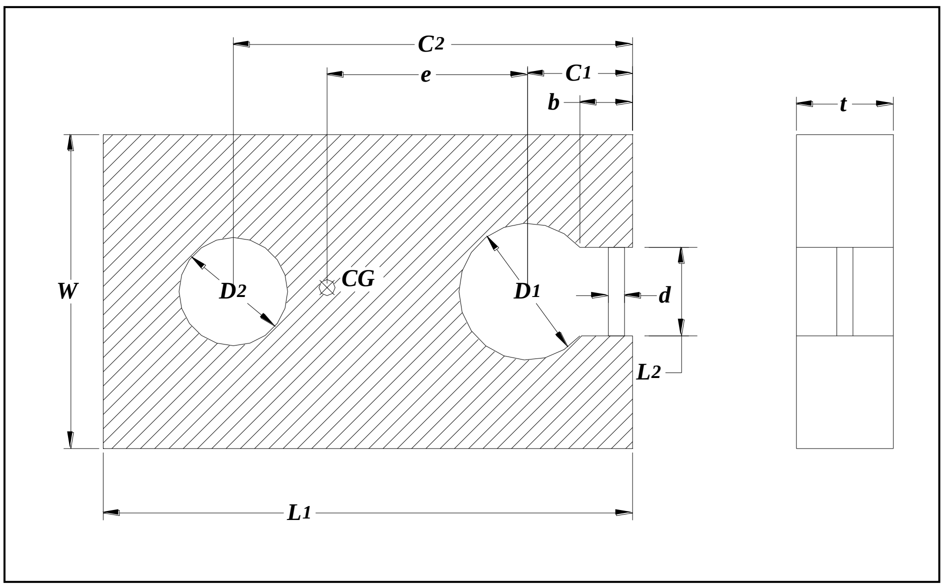


Figure-10.4 *Nomenclature of the blocks*

$$m = \rho\left(L_1 wt - \frac{\pi}{4} D_1^{\,2} t - \frac{1}{8} D_1^{\,2} t + \frac{\pi}{16} D_1^{\,2} t - \frac{\pi}{4} D_2^{\,2} t - bL_2 t + \frac{\pi}{4} d^2 L_2 \right) \qquad (9)$$

$$e=\frac{\rho}{m}\begin{pmatrix} L_1wt\left(\frac{L_1}{2}-C_1\right)-\left(\frac{\pi}{16}D_1{}^2t-\frac{1}{8}D_1{}^2t\right)(C_1-b)- \\ \frac{\pi}{4}D_2{}^2t(C_2-C_1)+\left(bL_2t-\frac{\pi}{4}d^2L_2\right)\left(C_1-\frac{b}{2}\right) \end{pmatrix} \qquad (10)$$

VI- Experimental Procedures:

1- Take all the dimensions and perform your calculations as will be demonstrated, and complete balancing process of the rotating shaft by finding the missing variables.
2- Fix the four blocks on the rotating shaft with the corresponding longitudinal distances from its end a_i, and the angular orientations θ, according to your balancing calculations.
3- Connect the shaft to the motor through the flexible belt.
4- Run the motor, and vary its speed to observe the vibrations of the system.

According to your calculations, this configuration of the four blocks on the shaft should give a balanced rotating system, and you can check it out from the behaviour of the system as it should not generate any vibrations, and rotates smoothly.
To differentiate the behaviour of a balanced system from an unbalanced one, you can disturb the configuration of the four blocks with respect to each other *(change a or/and θ)*, and rotate the shaft again, then notice the vibrations or fluctuations of the system.

VII- Collected Data:

Table-10.1 Basic dimensions of the four blocks

Differentiated Dimensions Among the Four Blocks				
Block	***(1)***	***(2)***	***(3)***	***(4)***
D_2 (mm)				
C_2 (mm)				

Shared Dimensions Among the Four Blocks			
Parameter	***Value***	***Parameter***	***Value***
D_1 (mm)		***C_1 (mm)***	
L_1 (mm)		***L_2 (mm)***	
t (mm)		***w (mm)***	
b (mm)		***d (mm)***	

Table-10.2 Data obtained concerning the first two blocks-1 & 2

Block	$\theta\,(^\circ)$	***S (mm)***

(1)		
(2)		

VIII- Data Processing:

- Use the dimensions measured, and apply in *eqns-9 & 10* to find m and e for each block.
- Determine the quantity me for the four blocks.
- Determine the quantity ame for blocks-1 & 2.
- Note:

$a_1 = 0 \Rightarrow a_1 m_1 e_1 = 0.$

- On a graph paper, draw to scale from the origin the vector m_1e_1 at the angle θ_1, and then continue from its tip with the vector m_2e_2 at angle θ_2.
- From the end of the second vector, draw a circle with radius m_3e_3, and from the origin draw a circle of radius m_4e_4.
- Join the intersection point of the two circles with the end of vector-2 to get vector-3, and join it with the origin to get vector-4.
- Measure the angles of the two vectors θ_3 and θ_4.
- On another graph paper, draw from the origin the vector $a_1m_1e_1$ at the angle θ_1, and then continue with $a_2m_2e_2$ at θ_2.
- From the end of the second vector, draw a line at angle θ_3, and from the origin another one at angle θ_4.
- The intersection of them identifies vectors-3 & 4, and their lengths are $a_3m_3e_3$ and $a_4m_4e_4$, respectively.

And so, you can find a_3 and a_4, then S_3 and S_4, according to your scale.

The previous method outlined is a graphical method, and you can obtain more accurate results by solving *eqns-5 & 6* simultaneously, to find θ_3 and θ_4, and then *eqns-7 & 8 to* get a_3 and a_4.

**** Note that:***

$a_i = S_i - S_1$**, as we have chosen plane-1 as the reference plane.**

IX- Results:

Table-10.3 *Data processing analysis*

Block	*m (kg)*	*e (mm)*	*me (kg.m)*	*ame (kg.m^2)*
(1)				
(2)				
(3)				-------------------
(4)				-------------------

Table-10.4 *Data processing results*

From the two graphs:			
Block	θ *(°)*	*ame (kg.m^2)*	*a (mm)*
(3)			
(4)			

X- Discussion And Conclusions:

1. **Name some practical examples in which balancing technique is necessary, and so employed?**

2. **For the disk mentioned in the example of static balancing technique, it was shown that it is statically balanced. Based on that description is it also dynamically balanced? Why?**

3. **It can be easily concluded that static balancing dose not imply dynamic balancing. Describe how can you check that with the system used in the experiment, after being balanced?**

4. **Could we consider static balancing technique an adequate alternative for dynamic balancing in some special cases? If yes, explain when and give a practical example?**

5. **You are given a build-in system that you cannot change its configuration; like a shaft loaded with parts of known eccentric masses, at fixed separating distances and with fixed angular orientations. How could you balance such a system?**

6. **Comment on your observations concerning the behaviour of the system, when you had tested your balancing calculations experimentally?**